Implicit Satisfaction of Battery Complementarity Constraints in Optimal Power Flow Problems

Abstract

Optimal power flow problems with battery energy storage are challenging when charging and discharging efficiencies are considered. One way to model this is to separate the charging and discharging powers of the battery and also enforce a nonconvex complementarity constraint. This nonconvexity is typically addressed through mixed-integer programming or a specialized solver which significantly increases computational complexity. In this paper, we prove that when battery inefficiency penalties are included in the objective function with positive cost coefficients, the complementarity constraint is implicitly satisfied at optimality. We establish this result for general convex optimization problems and demonstrate its application to multi-period DC microgrid optimal power flow. By combining our complementarity relaxation with standard semidefinite programming (SDP) techniques for voltage constraints, we obtain a fully convex formulation that can be solved efficiently.

Index Terms

Battery energy storage, DC network, microgrid, convex optimization, power systems optimization.

I. Introduction

The integration of battery energy storage into power grids requires solving optimal power flow (OPF) problems that coordinate generation, load, and storage across multiple time periods. When charging and discharging efficiencies are considered, a natural formulation separates net battery power into charging and discharging variables, requiring an explicit complementarity constraint to prevent simultaneous charging and discharging. This constraint is nonconvex and typically enforced via mixed-integer programming, significantly increasing computational complexity [1], [2].

SDP relaxations for AC [3] and DC [4] OPF have proven effective for voltage nonconvexities. Battery complementarity via penalization was studied in [5] for a specific AC linearization; our work gives an explicit penalty form valid for *any* convex OPF, with a novel SDP relaxation for DC grids.

As a motivating example, consider the single-bus system in Fig. 1: a 1 MW generator and a 0.5 MW battery serve a 1 MW load. Without the complementarity constraint, CVX yields the physically infeasible solution of charging 12.83 MW and discharging 13.33 MW simultaneously. Adding the constraint makes the problem nonconvex; instead, Theorem 1 shows that adding a small penalty (e.g., $p_c + p_d$) to the objective implicitly enforces complementarity while preserving convexity.

We prove that with positive penalty coefficients on charging and/or discharging, any optimal solution of the relaxed (complementarity-free) problem automatically satisfies $p_i^c(t) \cdot p_i^d(t) = 0$. We then combine this with SDP relaxation of DC voltage constraints, yielding a fully convex formulation validated on 304 test cases.

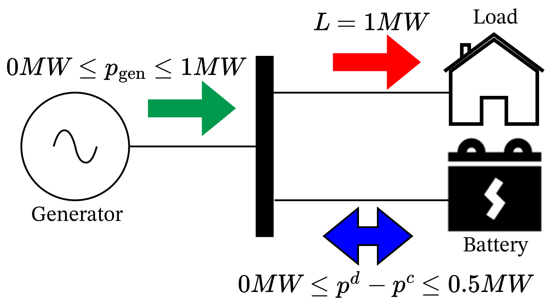

Fig. 1. Single-bus motivating example.

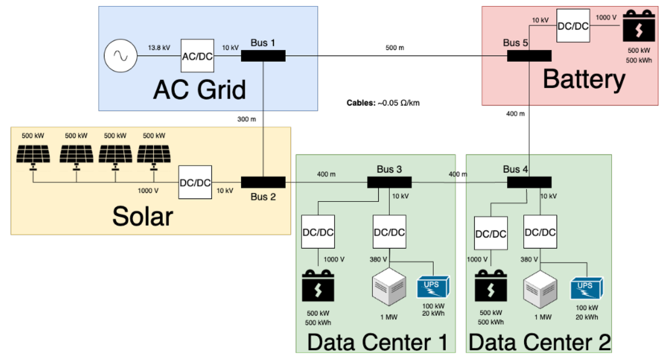

Fig. 2. 5-bus DC microgrid: datacenters (buses 3,4), solar (bus 2), batteries (buses 3,4,5), AC grid (bus 1).

II. Implicit Satisfaction of Complementarity Constraints

A. Problem Formulation

Let $\mathcal{T}$, $\mathcal{N}$, $\mathcal{B} \subseteq \mathcal{N}$ denote time steps, buses, and battery buses. For $i \in \mathcal{B}$, $t \in \mathcal{T}$: let $p_i^c(t), p_i^d(t) \geq 0$ be charging/discharging power, $E_i(t)$ stored energy, and x all remaining variables. We consider:

$$\min_z \; f_0(p^d - p^c, x) + \textstyle\sum_{t,i}\left[w_{c,i}(t)p_i^c(t) + w_{d,i}(t)p_i^d(t)\right] \tag{1a}$$

$$\text{s.t. } p_i^c(t) \cdot p_i^d(t) = 0 \tag{1b}$$

$$E_i(t+1) = E_i(t) - \tfrac{\Delta t}{\eta_d} p_i^d(t) + \eta_c \Delta t p_i^c(t) \tag{1c}$$

$$\underline{E_i} \leq E_i(t) \leq \overline{E_i},\; p_i^c, p_i^d \geq 0,\; f_j(z) \leq 0,\; h_j(z) = 0 \tag{1d}$$

Coefficients $w_{c,i}, w_{d,i} \geq 0$ encode efficiency losses, degradation, or prices. Constraint (1b) is the sole nonconvexity when f_0, f_j are convex and h_j affine; we show it is implicitly satisfied under mild conditions on these coefficients.

B. Main Result

Theorem 1. *Consider the optimization problem* (1) *with battery penalty function* (1a)*. If the following conditions hold:*

1) *The problem is convex when constraint* (1b) *is relaxed*
2) *For all $i \in \mathcal{B}$ and $t \in \mathcal{T}$, at least one of the following is satisfied:*
   a) $w_{c,i}(t) > 0, w_{d,i}(t) = 0$ *(charging is penalized)*
   b) $w_{c,i}(t) = 0, w_{d,i}(t) > 0$ *(discharging is penalized)*
   c) $w_{c,i}(t) > 0, w_{d,i}(t) > 0$ *(both are penalized)*

Then any optimal solution automatically satisfies the complementarity constraint:

$$p_i^c(t) \cdot p_i^d(t) = 0 \quad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{2}$$

We prove by contradiction. Full proof in the final paper.

C. Practical Implications

Theorem 1 removes the need to explicitly enforce (1b), making the problem solvable with standard convex solvers [6]. The result requires only that at least one of $w_{c,i}(t)$ or $w_{d,i}(t)$ is positive, a mild condition met by efficiency losses, time-of-use pricing, or battery degradation costs. If none of the conditions hold, the constraint must be enforced explicitly.

III. Application to DC Power Flow

We apply Theorem 1 to DC microgrid OPF, combining complementarity relaxation with SDP relaxation for voltage constraints to obtain a fully convex formulation.

A. Nonconvex Multi-period DC Power Flow Problem

We consider a true DC grid (real voltages and powers, no reactive power or voltage angles, not to be confused with the DC linearization of AC OPF), such as High Voltage DC (HVDC) lines or DC micrograms. The multi-period OPF is:

$$\min_{z} \quad f_0(p^d - p^c, x) + \textstyle\sum_{t,i} f_{\text{penalty},i}(t)$$

$$\text{s.t.} \quad p_i(t) - L_i(t) = v_i(t) \sum_{j \in \mathcal{N}} (v_i(t) - v_j(t)) y_{ij} \tag{3a}$$

$$y_{ij}^2 (v_i(t) - v_j(t))^2 \leq \overline{I}_{ij}^2, \ \ \underline{v}_i \leq v_i(t) \leq \overline{v}_i \tag{3b}$$

$$p_i^d(t) - p_i^c(t) = p_i(t), \ \underline{p}_i(t) \leq p_i(t) \leq \overline{p}_i(t), \tag{3c}$$

(1b)–(1d)

where y_{ij} is line conductance, $\overline{I}_{ij}$ is current limit, and $L_i(t)$ is load. Nonconvexities arise from bilinear voltages in (3a), quadratic currents in (3b), and (1b).

B. Convex Relaxation via Semidefinite Programming

1) SDP Lifting: Define the lifted variable $V(t) = v(t)v(t)^\top$, so $V_{ii}(t) = v_i^2(t)$ and $V_{ij}(t) = v_i(t)v_j(t)$. This linearizes the power balance, voltage bounds, and current limits:

$$p_i(t) - L_i(t) = \textstyle\sum_{j \in \mathcal{N}} (V_{ii}(t) - V_{ij}(t)) y_{ij} \tag{4}$$

$$\underline{v}_i^2 \leq V_{ii}(t) \leq \overline{v}_i^2 \tag{5}$$

$$y_{ij}^2 (V_{ii} - V_{ij} - V_{ji} + V_{jj}) \leq \overline{I}_{ij}^2 \tag{6}$$

The exact constraint $V(t) = v(t)v(t)^\top$ requires $V(t) \succeq 0$ and $\mathrm{rank}(V(t)) = 1$. The SDP relaxation drops the rank-1 constraint.

2) Full Convex Relaxation: Replacing the nonconvex constraints in (3) with their lifted versions (4)–(6) and adding $V(t) \succeq 0$ yields a convex SDP. By Theorem 1, any optimal solution of this SDP automatically satisfies complementarity provided the penalty conditions hold.

The resulting problem is a convex SDP, exact when $\mathrm{rank}(V^*(t)) = 1$ (tight SDP) and Theorem 1 holds, both verified in Section IV.

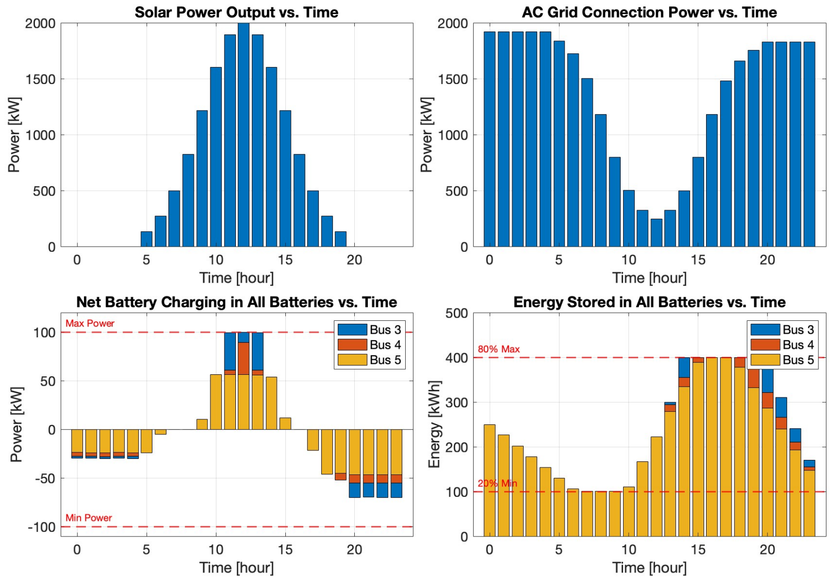

Fig. 3. Solar Profile and Power and Energy Dispatch Decisions over 24-hours. These dispatch decisions assume constant AC grid cost, constant data center demand, and initial battery state of charge at 50%.

IV. Numerical Studies

We validate on the 5-bus DC microgrid (Fig. 2), optimized over 24 hours ($T = 24$, $\Delta t = 1\,\text{hr}$) across 304 test cases varying initial SOC (5%–95%) and efficiency (80%–95%). Fig. 3 provides a visualization of one of these test cases. Bell-curve solar power output with fixed AC grid cost and constant data center demand. The battery energy is constrained to operate in the 20% to 80% region. The objective is to minimize the cumulative AC grid cost. The batteries start at 50% charged. At the start of the day, there is no sun, so the data center load is satisfied by a mixture of AC grid and battery power. As the sun rises, the solar power is able to help satisfy the load which reduces the AC grid usage and starts to charge the batteries. As the sun starts to set, solar power decreases and the AC grid and batteries satisfy the demand. Fig. 4 summarizes all four validation metrics across 304 test cases. *Complementarity (bottom right):* maximum $|p_i^c(t) \cdot p_i^d(t)| < 10^{-10}$, confirming Theorem 1. *Objective gap (bottom left):* average costs are 850.2421 (convex) vs. 850.2431 (nonconvex), with a maximum relative error of 0.00012%, confirming zero relaxation gap. *SDP exactness (top left):* maximum rank-1 deviation of $V^*(t)$ below 10^{-9}, confirming all voltage matrices are numerically rank-1. *Computation (top right):* median speedup is $\approx 8400\times$; the worst-case convex solution is still $93\times$ faster, demonstrating consistent suitability for real-time applications.

To assess real-world feasibility, the same 304 test cases are being executed on a Raspberry Pi embedded platform. The final paper will include a diagram of the hardware setup and a computation time box plot (analogous to Fig. 4, top right) comparing solver runtimes on the embedded device.

V. Conclusion

This paper demonstrated that the nonconvex complementarity constraint on battery charging/discharging can be relaxed without loss of feasibility when battery penalties are included in the objective. We established this

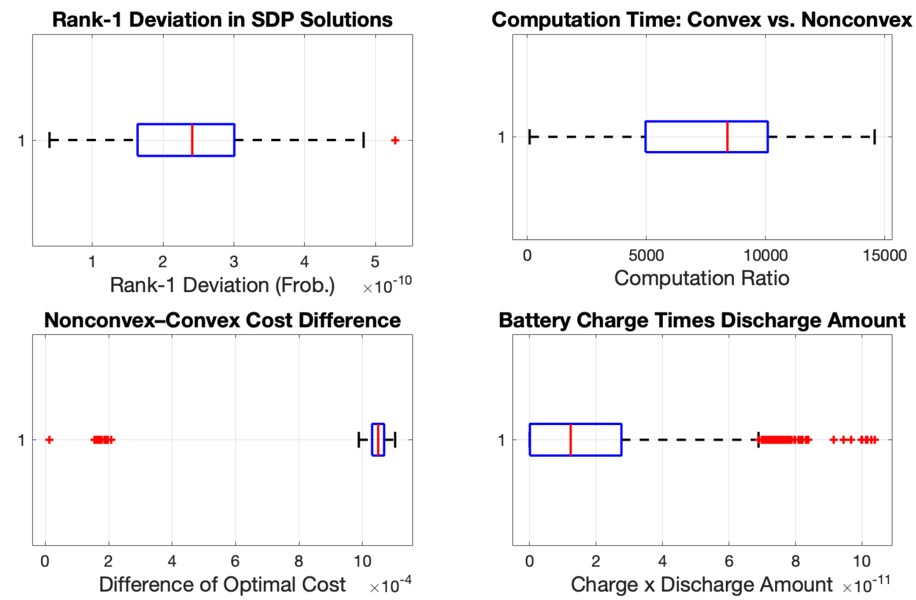

Fig. 4. Validation metrics over 304 test cases including Rank-1 deviation, optimal objective value differences (nonconvex minus convex), and magnitude of battery charge quantity multiplied by discharge quantity.

theoretically and validated it through 304 test cases on a 5-bus DC microgrid. The convex relaxation achieves 3 orders of magnitude computational speedup while maintaining solution quality, with all voltage matrices numerically rank-1 and zero complementarity violations. Hardware validation on an embedded Raspberry Pi platform is ongoing; full results will appear in the final paper. Future work includes extensions to AC OPF, distributed optimization for large battery fleets, and multi-timescale hierarchical control coordinating day-ahead scheduling with real-time dispatch.

References

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