

Implicit Satisfaction of Battery Complementarity Constraints in Optimal Power Flow Problems

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Abstract—Optimal power flow (OPF) problems with battery energy storage become challenging when charging and discharging efficiencies are considered because they introduce nonconvex constraints. One way to include charging and discharging efficiencies in the model is to separate the charging and discharging powers of the battery and also enforce a nonconvex complementarity constraint. This nonconvexity is typically addressed through mixed-integer programming or a specialized solver, which significantly increases computational complexity. In this paper, we prove that when battery inefficiency penalties are included in the objective function with positive cost coefficients, the complementarity constraint is implicitly satisfied at optimality. We establish this result for general convex OPF problems and demonstrate its application to multi-period dc microgrid optimal power flow. By combining our complementarity relaxation with standard semidefinite programming (SDP) techniques for voltage constraints, we obtain a fully convex formulation that can be solved efficiently. Because the formulation is convex, it runs on low-cost edge hardware. This enables local, real-time control without cloud infrastructure. We demonstrate this by deploying the multi-period dc microgrid OPF on a Raspberry Pi 5 using CVXPYgen-generated C code, with solve times orders of magnitude below the dispatch interval and solution quality matching a laptop solver.

Index Terms—Battery energy storage, dc network, microgrid, convex optimization, power systems optimization.

I. INTRODUCTION

The integration of battery energy storage into power grids requires solving optimal power flow (OPF) problems that coordinate generation, load, and storage across multiple time periods. When charging and discharging efficiencies are considered, a natural formulation separates net battery power into charging and discharging variables, requiring an explicit complementarity constraint to prevent simultaneous charging and discharging. This constraint is nonconvex and typically enforced via mixed-integer programming, significantly increasing computational complexity [1]–[3].

Semidefinite programming (SDP) [4] relaxations for ac [5]–[7] and dc OPF [8] have proven effective for voltage nonconvexities. Battery complementarity via penalization was studied in [9] for a linearized ac OPF formulation using Karush-Kuhn-Tucker conditions, and sufficient conditions for exact relaxation of complementarity constraints in general energy storage optimization problems were derived in [10]. However, [9] is limited to a specific linearized formulation

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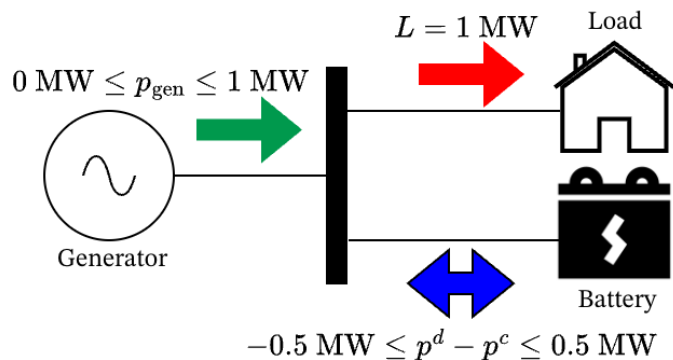


Fig. 1. Single-bus motivating example.

with a restricted cost structure, and does not establish the penalty approach for general convex power flow problems or apply to dc grids.

For a motivating example, consider the OPF problem for the single-bus system in Fig. 1. The 1 MW load must be satisfied by a generator with 1 MW capacity and a battery with 0.5 MW capacity. The objective is to minimize generator usage while satisfying the load. The optimal solution occurs when the battery provides 0.5 MW and the generator makes up the difference (0.5 MW). When the complementarity constraint is not enforced, there are infinitely many minimizers. For example, discharge 1.5 MW and charge 1 MW, or discharge 0.75 MW and charge 0.25 MW. When solving this problem using CVX, the solver returns the physically infeasible solution of charging 12.83 MW and discharging 13.33 MW simultaneously. Adding the complementarity constraint turns this simple convex problem into a nonconvex problem.

We prove that with positive penalty coefficients on charging and/or discharging, any optimal solution of the relaxed (complementarity-free) problem automatically satisfies $p_i^c(t) \cdot p_i^d(t) = 0$. We demonstrate this result in the context of dc microgrids, which have emerged as a promising architecture for integrating distributed energy resources such as solar generation and battery storage [11], [12], and where real-time energy management demands computationally efficient formulations [13], [14]. We then demonstrate the application of the proposed complementarity result to a dc-grid OPF problem by combining it with an SDP relaxation of the voltage constraints, yielding a fully convex formulation. Through

experiments on 315 test cases varying initial battery state of charge (SOC), photovoltaic (PV) scaling, and load scaling, we verify that: (a) the complementarity constraint is satisfied with zero violations, validating the theoretical predictions, and (b) the SDP relaxation of the power flow problem for dc grids is exact.

The main contributions of this paper, which extend [9], are: (1) an explicit penalty function that implicitly satisfies the complementarity constraint for *any* convex OPF problem; (2) a more general cost function that encompasses efficiency losses, degradation, and time-varying prices; (3) application to a dc grid with a novel SDP relaxation for voltage constraints; and (4) hardware validation on a Raspberry Pi 5 using CVXPYgen-generated C code, confirming real-time feasibility on embedded platforms.

II. IMPLICIT SATISFACTION OF COMPLEMENTARITY CONSTRAINTS

A. Problem Formulation

Let $\mathcal{T}$, $\mathcal{N}$, $\mathcal{B} \subseteq \mathcal{N}$ denote the sets of time steps, buses, and battery buses. For $i \in \mathcal{B}$, $t \in \mathcal{T}$: let $p_i^c(t), p_i^d(t) \geq 0$ be charging/discharging power, $E_i(t)$ stored energy with minimum and maximum limits $\underline{E}_i, \bar{E}_i$, and Δt time step. Let $x(t)$ denote the remaining decision variables at time t (e.g., generator outputs, controllable loads, bus voltages), with $x = (x(t))_{t \in \mathcal{T}}$ the stacked vector over all time steps. We require x 's defining constraints to depend on $p^c(t), p^d(t)$, if at all, only through the net difference $p^d(t) - p^c(t)$ – never individually; this holds for the voltage constraints in Section III, where power balance depends only on net injection $p_i(t) = p_i^d(t) - p_i^c(t)$, and excludes variables like $E(t)$, whose dynamics (1b) depend on $p^c(t)$ and $p^d(t)$ individually through the distinct efficiency coefficients $\eta_c, 1/\eta_d$; accordingly E is tracked separately rather than folded into x . We define $p^c(t) = [p_i^c(t)]_{i \in \mathcal{N}} \in \mathbb{R}^{|\mathcal{N}|}$ and $p^d(t) = [p_i^d(t)]_{i \in \mathcal{N}} \in \mathbb{R}^{|\mathcal{N}|}$ as the vectors of charging and discharging powers across all buses, with $p_i^c(t) = p_i^d(t) = 0$ for $i \notin \mathcal{B}$. Throughout, subscript i implies $\forall i \in \mathcal{N}$ and argument (t) implies $\forall t \in \mathcal{T}$ unless otherwise noted. We consider:

$$\min_z f_0(p^d - p^c, x) \quad (1a)$$

$$\text{s.t. } p_i^c(t) \cdot p_i^d(t) = 0 \quad (1a)$$

$$E_i(t+1) = E_i(t) - \frac{\Delta t}{\eta_d} p_i^d(t) + \eta_c \Delta t p_i^c(t) \quad (1b)$$

$$\underline{E}_i \leq E_i(t) \leq \bar{E}_i \quad (1c)$$

$$p_i^c(t) \geq 0, p_i^d(t) \geq 0 \quad (1d)$$

$$f_k(p^d - p^c, x) \leq 0, h_k(p^d - p^c, x) = 0 \quad (1e)$$

The objective f_0 can represent any convex cost, e.g., cost of generation or energy storage arbitrage profit [15]. Constraint (1a) is the sole nonconvexity when f_0, f_k are convex and h_k is affine; relaxing it yields a convex problem. As shown in the motivating example, simply dropping (1a) often produces

physically infeasible solutions. Instead, we modify (1) by adding a penalty term to the objective:

$$\min_z f_0(p^d - p^c, x) + \Delta t \sum_{t,i} [w_{c,i}(t) p_i^c(t) + w_{d,i}(t) p_i^d(t)] \quad (2)$$

where $w_{c,i}(t), w_{d,i}(t) \geq 0$ are cost coefficients encoding efficiency losses, degradation, or time-varying prices. We show that under mild conditions on these coefficients, (1a) is implicitly satisfied at any optimal solution of the relaxed (complementarity-free) version of (2).

TABLE I
NOMENCLATURE

Symbol	Description
Sets	
$\mathcal{T}$	Set of discrete time periods
$\mathcal{N}$	Set of buses in the microgrid
$\mathcal{B}$	Set of buses with batteries ($\mathcal{B} \subseteq \mathcal{N}$)
Variables	
$v_i(t)$	Voltage at bus i at time t
$p_i(t)$	Net real power injection at bus i at time t
$E_i(t)$	Energy stored in battery at bus i at time t
$p_i^d(t)$	Power discharged by battery at bus i at time t (scalar)
$p^d(t)$	Vector of discharging powers, $[p_i^d(t)]_{i \in \mathcal{N}} \in \mathbb{R}^{ \mathcal{N} }$
$p_i^c(t)$	Power charged by battery at bus i at time t (scalar)
$p^c(t)$	Vector of charging powers, $[p_i^c(t)]_{i \in \mathcal{N}} \in \mathbb{R}^{ \mathcal{N} }$
x	Remaining decision variables independent of p^c, p^d
Parameters	
$\underline{v}_i, \bar{v}_i$	Min., max. voltage at bus i
$\underline{p}_i(t), \bar{p}_i(t)$	Min., max. power injection at bus i at time t
$\underline{E}_i, \bar{E}_i$	Min., max. battery energy at bus i
$\bar{I}_{ij}$	Max. current magnitude on line $i \rightarrow j$
$L_i(t)$	Load demand at bus i at time t
y_{ij}	Conductance of line $i \rightarrow j$
Δt	Discrete time step size
η_c, η_d	Battery charging, discharging efficiency
$w_{c,i}(t), w_{d,i}(t)$	Charge, discharge cost coefficients at bus i , time t
c_i	Amortized battery degradation cost at bus i (\$/MWh)
$f_k(z), h_k(z)$	Inequality, equality constraints

B. Main Result

Theorem 1. Consider the optimization problem (1) with battery penalty function (2). If the following conditions hold:

- 1) The problem is feasible and attains its optimum,
- 2) When the constraint (1a) is dropped, the relaxed problem is convex and attains its optimum,
- 3) For all $i \in \mathcal{B}$ and $t \in \mathcal{T}$: $w_{c,i}(t) \geq 0$, $w_{d,i}(t) \geq 0$, and $w_{c,i}(t) + w_{d,i}(t) > 0$.
- 4) For all $i \in \mathcal{B}$ and $t \in \mathcal{T}$ with $p_i^c(t) > 0$ and $p_i^d(t) > 0$, replacing the charging and discharging powers with their net-equivalent single-direction values keeps the stored energy below its upper limit at all subsequent times $\tau > t$:

- Net discharging ($p_i^d(t) \geq p_i^c(t)$): $E_i(\tau) + \Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p_i^c(t) \leq \bar{E}_i$.
- Net charging ($p_i^c(t) > p_i^d(t)$): $E_i(\tau) + \Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p_i^d(t) \leq \bar{E}_i$.

Then any optimal solution automatically satisfies the complementarity constraint:

$$p_i^c(t) \cdot p_i^d(t) = 0 \quad \forall i \in \mathcal{B}, t \in \mathcal{T} \quad (3)$$

Proof. We prove by contradiction. By conditions 1 and 2, the relaxed problem (without (1a)) is feasible and attains its optimal value. Since it is also convex by condition 2, any optimal solution is a global optimum: no feasible point can achieve a strictly lower objective value. Suppose there exists an optimal solution $z^* = (p_i^{c*}, p_i^{d*}, \dots)$ and a time $\hat{t} \in \mathcal{T}$ and a battery $\hat{i} \in \mathcal{B}$ such that:

$$p_{\hat{i}}^{c*}(\hat{t}) > 0 \quad \text{and} \quad p_{\hat{i}}^{d*}(\hat{t}) > 0$$

We construct an alternative feasible solution $\hat{z}$ with strictly lower objective value, contradicting the global optimality of z^* . For notational simplicity, we drop the subscript $\hat{i}$ in the following analysis. We consider two subcases based on net power flow.

Subcase A: Net Discharging. Assume $p^{d*}(\hat{t}) \geq p^{c*}(\hat{t})$.

Define the modified solution $\hat{z}$ by setting:

$$\begin{aligned} \hat{p}^d(\hat{t}) &= p^{d*}(\hat{t}) - p^{c*}(\hat{t}), \quad \hat{p}^c(\hat{t}) = 0, \\ \hat{E}(\tau) &= E^*(\tau) + \Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p^{c*}(\hat{t}), \quad \forall \tau > \hat{t}, \end{aligned}$$

with all remaining variables unchanged from z^* .

Feasibility: The solution $\hat{z}$ satisfies all constraints because:

- 1) *Net power unchanged:* The net power $\hat{p}(\hat{t}) = \hat{p}^d(\hat{t}) - \hat{p}^c(\hat{t}) = p^{d*}(\hat{t}) - p^{c*}(\hat{t})$ remains the same
- 2) *Power limits:* Since $p^{d*}(\hat{t}) \geq p^{c*}(\hat{t}) \geq 0$, we have $\hat{p}^d(\hat{t}) \geq 0$, and the upper bound is satisfied: $\hat{p}^d(\hat{t}) \leq p^{d*}(\hat{t}) \leq \bar{p}^d$
- 3) *Battery energy limits:* The swap leaves net power unchanged but removes the round-trip loss, so by (1b) the stored energy rises by $\Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p^{c*}(\hat{t}) \geq 0$ at every $\tau > \hat{t}$. The lower limit therefore holds automatically, and the net-discharging case of condition 4 ensures the upper limit $\bar{E}$ is preserved.

Objective comparison: The penalty reduction is:

$$\begin{aligned} \Delta f_{\text{penalty}}(\hat{t}) &= w_c(\hat{t})p^{c*}(\hat{t}) + w_d(\hat{t})p^{d*}(\hat{t}) \\ &\quad - w_d(\hat{t})[p^{d*}(\hat{t}) - p^{c*}(\hat{t})] \\ &= [w_c(\hat{t}) + w_d(\hat{t})]p^{c*}(\hat{t}) > 0 \end{aligned}$$

where the strict inequality holds because $w_c(\hat{t}) + w_d(\hat{t}) > 0$ by condition 3 and $p^{c*}(\hat{t}) > 0$ by hypothesis.

Subcase B: Net Charging. Assume $p^{c*}(\hat{t}) > p^{d*}(\hat{t})$.

Instead define:

$$\begin{aligned} \hat{p}^c(\hat{t}) &= p^{c*}(\hat{t}) - p^{d*}(\hat{t}), \quad \hat{p}^d(\hat{t}) = 0, \\ \hat{E}(\tau) &= E^*(\tau) + \Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p^{d*}(\hat{t}), \quad \forall \tau > \hat{t}, \end{aligned}$$

with all remaining variables unchanged from z^* . By an analogous argument, net power is unchanged, the stored energy again rises by $\Delta t \left(\frac{1}{\eta_d} - \eta_c \right) p^{d*}(\hat{t}) \geq 0$ for all $\tau > \hat{t}$, so the lower limit holds automatically and the net-charging case of

condition 4 ensures the upper limit is preserved. The penalty reduction is:

$$\Delta f_{\text{penalty}}(\hat{t}) = [w_c(\hat{t}) + w_d(\hat{t})]p^{d*}(\hat{t}) > 0,$$

again strictly positive since $w_c(\hat{t}) + w_d(\hat{t}) > 0$ and $p^{d*}(\hat{t}) > 0$.

In both subcases, f_0 evaluates identically at z^* and $\hat{z}$ since net battery power $p^d - p^c$ is unchanged. Therefore the total objective strictly decreases, contradicting optimality of z^* . We conclude that at any optimal solution $p_i^c(t) \cdot p_i^d(t) = 0$ for all $i \in \mathcal{B}, t \in \mathcal{T}$. $\square$

C. Physical Battery Loss Example

A natural choice of penalty coefficients arises from efficiency losses. During charging, only a fraction $\eta_c \in (0, 1)$ of the input power $p_i^c(t)$ is actually stored in the battery. The remaining $(1 - \eta_c)p_i^c(t)$ is dissipated as heat. During discharging, to deliver $p_i^d(t)$ to the grid the battery must internally release $p_i^d(t)/\eta_d$, so the power lost to inefficiency is $p_i^d(t)/\eta_d - p_i^d(t) = \frac{1-\eta_d}{\eta_d}p_i^d(t)$. Setting $w_{c,i}(t) = 1 - \eta_c$ and $w_{d,i}(t) = (1 - \eta_d)/\eta_d$ therefore makes the penalty term in (2) equal to the total power dissipated by inefficiency:

$$f_{\text{penalty},i}(t) = (1 - \eta_c)p_i^c(t) + \frac{1 - \eta_d}{\eta_d}p_i^d(t).$$

Since $\eta_c, \eta_d \in (0, 1)$, both coefficients are strictly positive, satisfying the conditions of Theorem 1.

D. Practical Implications

Theorem 1 removes the need to explicitly enforce (1a), making the problem solvable with standard convex solvers [16]. The result requires only that at least one of $w_{c,i}(t)$ or $w_{d,i}(t)$ is positive, a mild condition met by efficiency losses, time-of-use pricing, or battery degradation costs. If none of the conditions hold, the constraint must be enforced explicitly.

Remark 1. The theorem does not require both w_c and w_d to be positive. It suffices that at least one is positive at each time period. This allows for scenarios where:

- Only charging incurs costs (e.g., time-of-use pricing where discharging has zero marginal cost)
- Only discharging incurs costs (e.g., battery degradation primarily from discharge cycles)
- Both incur costs (e.g., the physical efficiency loss case)

We demonstrate how this complementarity relaxation can be combined with standard SDP relaxations for voltage constraints in OPF for dc grids. While we focus on dc networks for concreteness, the complementarity result applies equally to ac OPF when combined with appropriate convex relaxations of the ac power flow equations. The resulting formulation avoids both voltage bilinearities and complementarity nonconvexity, enabling efficient solving via standard convex solvers.

III. APPLICATION TO DC GRID OPTIMAL POWER FLOW

We apply Theorem 1 to dc grid OPF, combining complementarity relaxation with SDP relaxation for voltage constraints to obtain a fully convex formulation.

A. Nonconvex Multi-period dc Power Flow Problem

We consider a dc grid (with real voltages and powers and no reactive power or voltage angles, not to be confused with the dc linearization of ac OPF), such as High Voltage dc (HVdc) networks or dc microgrids. The multi-period OPF is:

$$\begin{aligned} \min_z \quad & f_0(p^d - p^c, x) \\ \text{s.t.} \quad & p_i(t) - L_i(t) = v_i(t) \sum_{j \in \mathcal{N}} (v_i(t) - v_j(t)) y_{ij} \quad (4a) \\ & \underline{p}_i(t) \leq p_i(t) \leq \bar{p}_i(t) \quad (4b) \\ & \underline{v}_i \leq v_i(t) \leq \bar{v}_i \quad (4c) \\ & y_{ij}^2 (v_i(t) - v_j(t))^2 \leq \bar{I}_{ij}^2, \quad i \rightarrow j \quad (4d) \\ & p_i^d(t) - p_i^c(t) = p_i(t), \quad (4e) \\ & (1a)-(1c) \end{aligned}$$

where $v_i(t)$ is bus voltage, $p_i(t)$ is net power injection, $L_i(t)$ is load, y_{ij} is line conductance, and $\bar{I}_{ij}$ is line current limit. Constraint (4a) enforces power balance via Ohm's law. The notation $i \rightarrow j$ denotes that bus i is connected to bus j with $i < j$. Problem (4) is nonconvex due to the bilinear voltage terms in the power balance (4a), the quadratic current limit constraint (4d), and the complementarity constraint (1a).

B. Convex Relaxation via Semidefinite Programming

We apply two relaxation techniques simultaneously: (1) the SDP relaxation for voltage nonconvexities, and (2) the complementarity relaxation from Theorem 1.

1) *SDP Lifting*: Define the lifted variable $V(t) = v(t)v(t)^\top$, so $V_{ii}(t) = v_i^2(t)$ and $V_{ij}(t) = v_i(t)v_j(t)$. This linearizes the power balance, voltage bounds, and current limits:

$$p_i(t) - L_i(t) = \sum_{j \in \mathcal{N}} (V_{ii}(t) - V_{ij}(t)) y_{ij} \quad (5)$$

$$\underline{v}_i^2 \leq V_{ii}(t) \leq \bar{v}_i^2 \quad (6)$$

$$y_{ij}^2 (V_{ii} - V_{ij} - V_{ji} + V_{jj}) \leq \bar{I}_{ij}^2 \quad (7)$$

The exact constraint $V(t) = v(t)v(t)^\top$ requires $V(t)$ to be positive semidefinite (denoted $V(t) \succeq 0$) and $\text{rank}(V(t)) = 1$ [16]. The SDP relaxation drops the rank-1 equality constraint because it is nonconvex. After obtaining the optimal solution of the relaxed problem, the rank-1 equality constraint is verified a posteriori to confirm exactness of the relaxation. If $\text{rank}(V^*(t)) > 1$, the SDP solution cannot be mapped back to a physical voltage vector and is therefore meaningless. However, if $\text{rank}(V^*(t)) = 1$, then the relaxation is exact and the voltage vector $v(t)$ can be recovered directly.

2) *Full Convex Relaxation*: The nonconvex problem (4) is relaxed by: (1) replacing the complementarity constraint (1a) with the penalty objective from (2), (2) replacing the nonconvex voltage constraints with their lifted versions (5)–(7), and (3) adding $V(t) \succeq 0$. The result is a convex SDP. By Theorem 1, any optimal solution automatically satisfies complementarity provided the penalty conditions hold.

The resulting problem is a convex SDP, exact when $\text{rank}(V^*(t)) = 1$ (tight SDP) and Theorem 1 holds, both verified in Section IV.

C. Properties of the Relaxation

The relaxation is exact when two conditions hold simultaneously: (1) the SDP relaxation is tight, meaning $\text{rank}(V^*(t)) = 1$ for all $t \in \mathcal{T}$ at the optimal solution, and (2) the penalty coefficients satisfy the conditions of Theorem 1. When both conditions hold, the optimal solution satisfies all nonconvex constraints of the original problem.

IV. NUMERICAL STUDIES

We validate the effectiveness and computational performance of the proposed convex relaxation on the 5-bus dc microgrid in Fig. 2. While this case study uses data centers as the load model, the proposed approach is general and applies to any dc grid with battery storage (and more broadly to ac grids when combined with appropriate convex relaxations of the ac power flow equations). The data center loads at buses 3 and 4 must be satisfied by a mixture of the solar output at bus 2, the batteries at buses 3, 4, and 5, and the ac grid at bus 1. The grid operator must decide when to charge and discharge the batteries as well as how much power to draw from the ac grid, while maintaining stable voltage and current levels. The line current limit is $\bar{I}_{ij} = 500$ A for all lines. Bus 1 is the slack bus with a fixed voltage of 10 kV, while all other buses have voltage limits of $10 \text{ kV} \pm 500 \text{ V}$. This grid uses ac/dc and dc/dc power converters to step up/down voltages to the baseline 10 kV. These converters are modeled as ideal and lossless throughout.

Ultimately, the grid operator wants to satisfy the load while minimizing costs. The ac grid has an explicit cost via the electricity price (here assumed to be a constant \$30/MWh). However, the grid operator must also account for battery costs. Battery degradation is modeled by adding a cost term $c_i p_i^d(t)$ to the objective, where c_i is the amortized degradation cost per unit energy and $p_i^d(t)$ is the discharging power. Following the leveled cost of storage framework of [17], we adopt a simplified, undiscounted form in which the amortized cost is computed as the capital replacement cost divided by the total energy throughput over the battery's lifetime:

$$c_i = \frac{\text{Capital cost (per kWh)}}{\text{Cycle life} \times \text{DOD} \times \eta_{rt}}.$$

Assuming a capital cost of \$300/kWh [18], a cycle life of 5000 cycles, 80% depth of discharge (DOD), and 90% round-trip efficiency η_{rt} :

$$c_i = \frac{300}{5000 \times 0.8 \times 0.9} \approx \$83.33/\text{MWh}.$$

This value sets the discharge penalty coefficient $w_{d,i}(t) = c_i = \$83.33/\text{MWh} \forall i \in \mathcal{B}$ in the objective (2).

Figs. 3–6 show one representative case, optimized over 24 hours ($T = 24$, $\Delta t = 1$ hr) with initial battery SOC at 50% and charging and discharging efficiencies $\eta_c = \eta_d = 90\%$ for all batteries. The battery energy is constrained to the 20%–80%

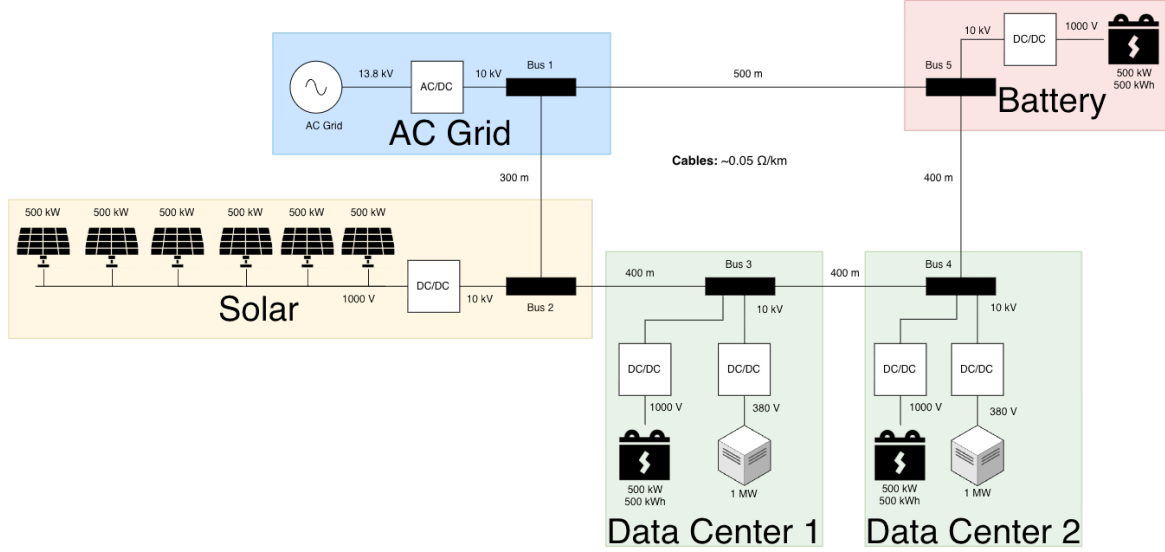


Fig. 2. 5-bus dc microgrid: data centers (buses 3,4), solar (bus 2), batteries (buses 3,4,5), ac grid (bus 1).

region. Fig. 3 shows the bell-curve solar output throughout the day, peaking at noon. Fig. 4 shows the ac grid usage throughout the day. Figs. 5 and 6 show the battery behavior. Fig. 5 shows the power charged and discharged by each battery throughout the day. Fig. 6 shows the energy in each battery throughout the day. At the start of the day, there is no sun, so load is met by a mix of ac grid and battery power. As the sun rises, solar reduces ac grid usage and charges the batteries. There is enough solar in the middle of the day to reduce the ac grid usage to zero and to charge the batteries to their maximum of 80%. As the sun sets, the grid and batteries again share the load. Note that the battery state of energy hits the minimum and maximum values throughout the day, but the battery power remains well within the limit of 500 kW. For this representative case, the complementarity product $|p_i^c(t) \cdot p_i^d(t)|$ was numerically zero across all batteries and time steps, and each optimal voltage matrix $V^*(t)$ was numerically rank-1, confirming both the complementarity relaxation and the exactness of the SDP relaxation for this scenario.

To assess robustness, we sweep over 315 test cases (7 initial SOC levels $\times$ 5 PV scaling factors $\times$ 9 load scaling factors) with battery efficiency fixed at 90%, initial SOC varying from 20%–80% in 10% increments, PV output scaled over 0.8–1.2, and load scaled over 0.5–1.3.

A. Implicit Complementarity Constraint Satisfaction

Across all 315 cases, the maximum observed $|p_i^c(t) \cdot p_i^d(t)|$ was 1.17×10^{-7} per unit (p.u.)² with a median of 8.80×10^{-12} p.u.², confirming that batteries never simultaneously charge and discharge and validating Theorem 1.

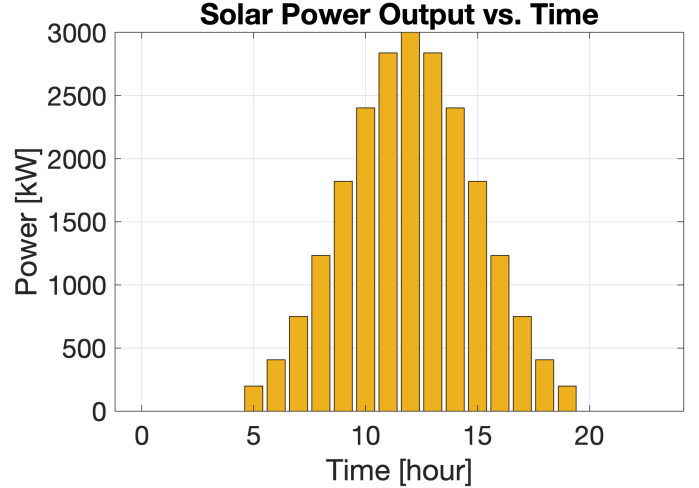


Fig. 3. Solar power output vs. time over 24 hours.

B. Exactness of Convex Relaxation

1) *Objective Value Comparison:* Across the 315 test cases, the operating cost of the convex relaxation differs from that of the nonconvex problem by a median of \$0.0024 (0.00025%) and at most \$0.19 (0.017%) in absolute value, confirming a negligible relaxation gap.

2) *Rank-1 Property of Voltage Matrices:* To verify exactness of the SDP relaxation, we compute the rank deficiency of the optimal voltage matrices $V^*(t)$ using singular value decomposition (SVD). Let σ_1 and u_1, v_1 denote the leading singular value and singular vectors of $V^*(t)$. The rank-1 approximation is $\sigma_1 u_1 v_1^T$, and the rank-1 deviation is defined

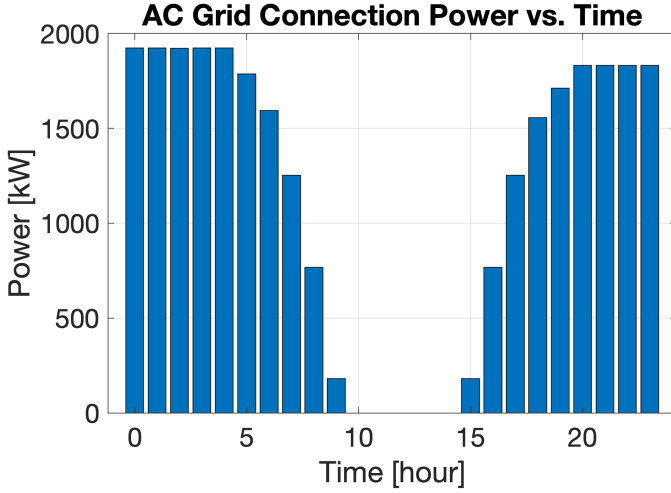


Fig. 4. ac grid connection power vs. time over 24 hours.

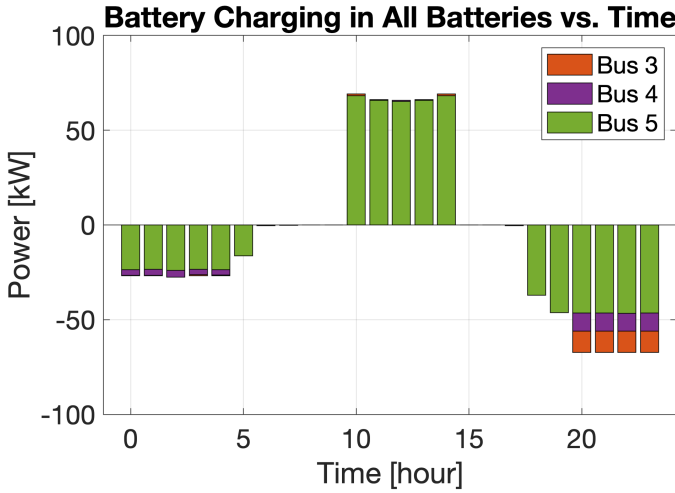


Fig. 5. Battery charging power across all batteries vs. time over 24 hours.

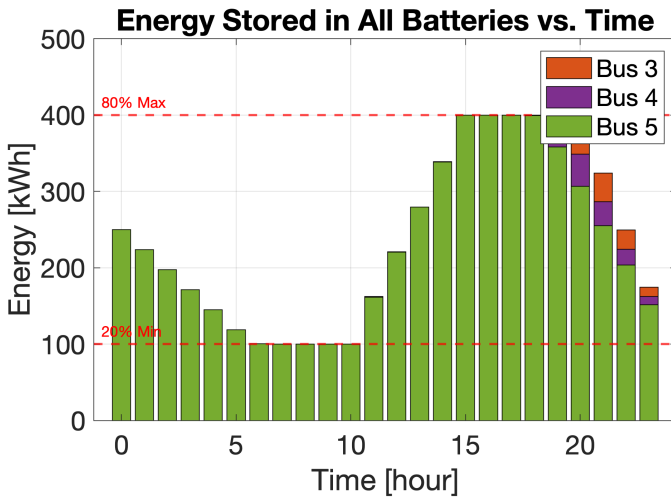


Fig. 6. Energy stored in all batteries vs. time over 24 hours.

TABLE II
MATLAB SIMULATION STATISTICS ACROSS 315 TEST CASES.
NC = NONCONVEX; C = CONVEX

Metric	Median	Max.
$ p_i^c - p_i^d $ (p.u. ²)	8.80×10^{-12}	1.17×10^{-7}
Abs. obj. diff. (\$)	2.41×10^{-3}	1.89×10^{-1}
Rank-1 deviation (p.u. ²)	2.20×10^{-10}	7.36×10^{-9}
Speedup ratio ($\times$)	3644	430 [†]

[†]Minimum observed value (worst-case speedup).

as $\|V^*(t) - \sigma_1 u_1 v_1^T\|_F$ [19]. Across all 315 test cases, the maximum rank-1 deviation was 7.36×10^{-9} p.u.² with a median of 2.20×10^{-10} p.u.², confirming all voltage matrices are numerically rank-1.

C. Computational Performance

All simulations were performed on an Apple MacBook Air with an Apple M1 chip (8-core, 4 performance + 4 efficiency), 8 GB RAM, running macOS 15.6.1, using MATLAB R2024b. The convex SDP was solved via CVX with the SDPT3 solver. The nonconvex problem was solved using the interior-point algorithm provided by the MATLAB Optimization Toolbox. The convex problem structure enables dramatically faster solution times. The median speedup is 3644 $\times$, and the worst-case convex solution is still 430 $\times$ faster, demonstrating that this advantage is consistent across diverse operating conditions and the approach is suitable for real-time applications. Table II summarizes all MATLAB simulation results.

D. Embedded Platform Validation

Our case study uses data centers, but the proposed method targets general dc grids, including remote or isolated ones with limited on-site compute and unreliable cloud access. To demonstrate feasibility in such settings, we run the same 315 test cases on a Raspberry Pi 5, a representative low-cost embedded controller. This is a proof of concept that the convex OPF can be solved on-site on constrained hardware, without relying on centralized cloud infrastructure.

1) *CVXPYgen Code Generation*: We use CVXPYgen [20] to deploy the convex SDP on the Raspberry Pi. CVXPYgen takes a CVXPY problem and automatically generates a lightweight, problem-specific C solver using the Clarabel interior-point solver [21]. Compared to running CVXPY directly on the Pi, the generated C code is significantly faster. Importantly, the generated C code can also run on more constrained microcontrollers that cannot execute Python, such as the Texas Instruments C2000 series [22], a family of real-time microcontrollers widely used for power-electronic converter control. This extends the approach to a wider range of embedded platforms.

2) *Client/Server Setup*: The Raspberry Pi 5 (Broadcom BCM2712, quad-core 64-bit Arm Cortex-A76 @ 2.4 GHz, 4 GB RAM, Debian GNU/Linux 13 (Trixie)) acts as the compute server, connected directly to the MacBook Air via a single Ethernet cable. The laptop dispatches problem data to the Pi, which solves the problem and measures solve time.

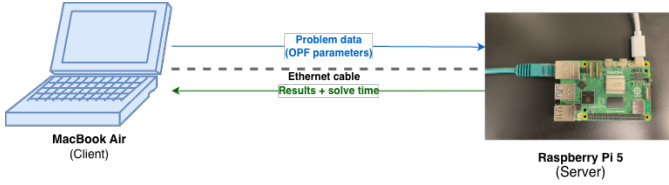


Fig. 7. Client/server setup: MacBook Air (client) connected to Raspberry Pi 5 (server) via Ethernet. The laptop sends problem data, the Pi solves, and results are returned over the local link.

TABLE III

RASPBERRY PI 5 SIMULATION STATISTICS ACROSS 315 TEST CASES. OBJ. DIFF. IS CVXPYGEN (RPi 5) MINUS CVX (LAPTOP)

Metric	Median	Max.
Solve time (s)	0.07	<0.1
Obj. diff. (%)	2.20×10^{-9}	5.58×10^{-2}
Rank-1 deviation (p.u. ²)	9.97×10^{-9}	1.32×10^{-6}
$ p_i^c \cdot p_i^d $ (p.u. ²)	3.79×10^{-12}	1.32×10^{-9}

Results and timing are returned to the laptop and plotted in MATLAB. Fig. 7 illustrates this architecture.

3) *Computation Time*: The median solve time for the convex problem on the Raspberry Pi 5 is 0.07 s, with a maximum below 0.1 s.

4) *Solution Quality on Embedded Platform*: To confirm that solution quality is preserved on the Pi, we compare the optimal objective value, rank-1 deviation, and complementarity product against the laptop results across all 315 test cases. The largest percentage difference in optimal objective values between CVXPYgen (Pi) and CVX (laptop) is 0.0558% with a median of $2.2 \times 10^{-9}\%$, confirming that both solvers converge to the same optimal point. The maximum rank-1 deviation on the Pi is 1.32×10^{-6} p.u.² with a median of 9.97×10^{-9} p.u.², consistent with the laptop results. The maximum observed $|p_i^c(t) \cdot p_i^d(t)|$ is 1.32×10^{-9} p.u.² with a median of 3.79×10^{-12} p.u.², confirming implicit complementarity satisfaction on the embedded platform. Table III summarizes all Raspberry Pi 5 results.

V. CONCLUSION

This paper proved that the nonconvex complementarity constraint on battery charging and discharging is implicitly satisfied at optimality when positive penalty coefficients are included in the objective, eliminating the need for mixed-integer programming. We further showed that combining this complementarity relaxation with an SDP relaxation of dc voltage constraints yields a fully convex formulation whose SDP relaxation is exact, confirmed by numerically rank-1 voltage matrices across all 315 test cases. The convex formulation achieves a median speedup of $3644\times$ over the nonconvex problem, with a worst-case speedup of $430\times$. Hardware validation on a Raspberry Pi 5 confirms that the convex OPF can be solved on an edge computing device in under 0.1 s with solution quality matching a laptop solver. Future work includes extension to ac OPF using appropriate convex relaxations, incorporation of non-ideal converter models and

more sophisticated battery degradation models, and distributed optimization for large-scale battery fleets.

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